

Covariant Quantization with Extended BRST Symmetry

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Abstract

A short review of covariant quantization methods based on BRST-antiBRST symmetry is given. In particular problems of correct definition of $Sp(2)$ symmetric quantization scheme known as triplectic quantization are considered.

1. Introduction

It is well-known that all the fundamental interactions (electromagnetic, strong, weak and gravitational) can be described in terms of gauge theories [1]. The quantization of gauge theories is one of the most essential means to investigate the quantum properties of the fundamental forces. The formalisms of Hamiltonian (or canonical) and Lagrangian (or covariant) quantization of gauge theories present two different approaches to the quantum description of dynamical systems [2].

The Lagrangian quantization remains one of the most attractive approaches to gauge theories quantization owing to its main advantage – the possibility of direct construction of the quantum effective action – allowing to avoid the usual long way of canonical quantization with subsequent integration over momenta in the path integral, producing as a result the vacuum expectation value of the S-matrix in the presence of external sources. During last years covariant quantization schemes for arbitrary gauge theories are intensively developed. The so-called $Sp(2)$ -covariant [3], superfield [4], triplectic [5] quantization methods have been discovered. In addition, a further extension to an $osp(1,2)$ -symmetric quantization [6], allowing also for a superfield formulation [7], has been found. These approaches are based on the principle of invariance under global BRST-antiBRST symmetry [8, 9, 10, 11] and are connected with different off-shell realizations of this invariance principle in construction of Green's functions.

The purpose of this paper is to introduce the reader into the $Sp(2)$ -covariant [3] and triplectic [5] quantization methods and to present a modified scheme of triplectic quantization [12].

2. $Sp(2)$ -covariant quantization

Let us consider the $Sp(2)$ -quantization for general gauge theory described by the initial classical action $S_0(A)$ of fields A^i with Grassmann parities $\epsilon(A^i) = \epsilon_i$. We assume that

$S_0(A)$ is invariant under gauge transformations $\delta A^i = R_\alpha^i(A)\xi^\alpha$,

$$S_{0,i}(A)R_\alpha^i(A) = 0, \quad (1)$$

where ξ^α are arbitrary functions, and $R_\alpha^i(A)$ are generators of gauge transformations. We have also used DeWitt's condensed notations [13], where any index includes all particular ones (space - time, index of internal group, Lorentz index and so on). Summation over repeated indices implies integration over continuous ones and usual summation over discrete ones. We assume that the set $\{R_\alpha^i(A)\}$ is complete. As a consequence of the condition of completeness, one can prove [14] that the algebra of generators has the following general form,

$$R_{\alpha,j}^i(A)R_{\beta}^j(A) - (-1)^{\varepsilon_\alpha\varepsilon_\beta}R_{\beta,j}^i(A)R_\alpha^j(A) = -R_\gamma^i(A)F_{\alpha\beta}^\gamma(A) - S_{0,j}(A)M_{\alpha\beta}^{ij}(A), \quad (2)$$

where $F_{\alpha\beta}^\gamma(A)$ are structure functions depending, in general, on the fields A^i with the following properties of symmetry $F_{\alpha\beta}^\gamma(A) = -(-1)^{\varepsilon_\alpha\varepsilon_\beta}F_{\beta\alpha}^\gamma(A)$ and $M_{\alpha\beta}^{ij}(A)$ satisfies the conditions $M_{\alpha\beta}^{ij}(A) = -(-1)^{\varepsilon_i\varepsilon_j}M_{\alpha\beta}^{ji}(A) = -(-1)^{\varepsilon_\alpha\varepsilon_\beta}M_{\beta\alpha}^{ij}(A)$.

Then it is necessary to introduce the total configuration space ϕ^A ,

$$\phi^A = (A^i, B^\alpha, C^{aa}, \dots), \quad \varepsilon(\phi^A) = \varepsilon_A. \quad (3)$$

Here, C^{aa} are the $Sp(2)$ -doublet of ghost ($a = 1$) and antighost ($a = 2$) fields with respect to the index a , B^α are auxiliary fields and dots denote (for reducible theories) pyramids of ghosts, antighosts and Lagrange multipliers which are combined into irreducible representations of the symplectic group $Sp(2)$ (see [3]). Note that the general ingredients and formulas have the same form both for irreducible and reducible cases.

To each field ϕ^A of the total configuration space one introduces three sets of anti-fields ϕ_{Aa}^* , $\varepsilon(\phi_{Aa}^*) = \varepsilon_A + 1$ and $\bar{\phi}_A$, $\varepsilon(\bar{\phi}_A) = \varepsilon_A$, being sources of extended BRST transformations, namely, BRST-transformations, antiBRST-transformations and mixed transformations respectively.

On the space of fields ϕ^A and antifields ϕ_{Aa}^* one defines [3] odd symplectic structures $(,)^a$, called the extended antibrackets

$$(F, G)^a \equiv \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi_{Aa}^*} - (F \leftrightarrow G) (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}. \quad (4)$$

The derivatives with respect to fields are understood as acting from the right and those with respect to antifields, as acting from the left. There are the graded Jacobi identities for the extended antibrackets:

$$((F, G)^{\{a}, H)^{b\}}(-1)^{(\varepsilon(F)+1)(\varepsilon(H)+1)} + \text{cycl.perm.}(F, G, H) \equiv 0, \quad (5)$$

where curly brackets denote symmetrization with respect to the indices a, b of the $Sp(2)$ group.

In addition the operators V^a , Δ^a are introduced

$$V^a = \varepsilon^{ab} \phi_{Ab}^* \frac{\delta}{\delta \phi_A}, \quad (6)$$

$$\Delta^a = (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi_{Aa}^*}, \quad (7)$$

where ε^{ab} is the antisymmetric tensor for raising and lowering $Sp(2)$ -indices

$$\varepsilon^{ab} = -\varepsilon^{ba}, \quad \varepsilon^{12} = 1 \quad \varepsilon_{ab} = -\varepsilon^{ab}.$$

It can be readily established that the algebra of the operators (6), (7) has the form

$$\Delta^{\{a}\Delta^{b\}} = 0, \quad \Delta^{\{a}V^{b\}} + V^{\{a}\Delta^{b\}} = 0, \quad V^{\{a}V^{b\}} = 0. \quad (8)$$

It is advantageous to introduce an operator $\bar{\Delta}^a = \Delta^a + (i/\hbar)V^a$ with the properties

$$\bar{\Delta}^{\{a}\bar{\Delta}^{b\}} = 0. \quad (9)$$

For a boson functional $S = S(\phi, \phi^*, \bar{\phi})$, we introduce the extended quantum master equations

$$\frac{1}{2}(S, S)^a + V^a S = i\hbar\Delta^a S \quad (10)$$

with the boundary condition

$$S|_{\phi^*=\bar{\phi}=\hbar=0} = S_0(A). \quad (11)$$

The generating equation for the bosonic functional S is a set of two equations. It should be verified that these equations are compatible. The simplest way to establish this fact is to rewrite the extended master equations in the equivalent form of linear differential equations

$$\bar{\Delta}^a \exp \left\{ \frac{i}{\hbar} S \right\} = 0. \quad (12)$$

Due to the properties (9) of the operators $\bar{\Delta}^a$, we immediately establish the compatibility of the equations.

With the help of action (10) we next define the vacuum functional $Z(0)$ by the rule

$$\begin{aligned} Z(0) = & \int d\phi d\phi^* d\bar{\phi} d\lambda d\pi^a \exp \left\{ \frac{i}{\hbar} \left(S(\phi, \phi^*, \bar{\phi}) + \phi_{Aa}^* \pi^{Aa} + \right. \right. \\ & \left. \left. + \left(\bar{\phi}_A - \frac{\delta F}{\delta \phi^A} \right) \lambda^A - \frac{1}{2} \varepsilon_{ab} \pi^{Aa} \frac{\delta^2 F}{\delta \phi^A \delta \phi^B} \pi^{Bb} \right) \right\}, \end{aligned} \quad (13)$$

where we have introduced a set of auxiliary fields $\pi^{Aa}, \varepsilon(\pi^{Aa}) = \varepsilon_A + 1, \lambda^A, \varepsilon(\lambda^A) = \varepsilon_A$ and a bosonic functional $F = F(\phi)$ fixing a gauge in the theory.

An important property of the integrand in (13) is its invariance under the following global transformations (which, for its part, is a consequence of the extended master equation for S)

$$\begin{aligned} \delta\phi^A &= \pi^{Aa}\mu_a, \quad \delta\phi_{Aa}^* = \mu_a \frac{\delta S}{\delta \phi^A}, \quad \delta\bar{\phi}_A = \varepsilon^{ab}\mu_a \phi_{Ab}^*, \\ \delta\pi^{Aa} &= -\varepsilon^{ab}\lambda^A \mu_b, \quad \delta\lambda^A = 0, \end{aligned} \quad (14)$$

where μ_a is an $\text{Sp}(2)$ -doublet of constant anticommuting Grassmann parameters. These transformations realize the extended BRST transformations in the space of the variables $\phi, \phi^*, \bar{\phi}, \pi$ and λ .

The existence of these transformations enables one to establish the independence of the vacuum functional from the choice of gauge. Indeed, suppose $Z_F \equiv Z(0)$. We shall change the gauge $F \rightarrow F + \Delta F$. In the functional integral for $Z_{F+\Delta F}$ we make the above-mentioned change of variables with the parameters chosen as

$$\mu_a = \frac{i}{2\hbar} \varepsilon_{ab} \frac{\delta \Delta F}{\delta \phi^A} \pi^{Ab}. \quad (15)$$

Then we find

$$Z_F = Z_{F+\Delta F} \quad (16)$$

and therefore the S -matrix is gauge-independent by virtue of the equivalence theorem [15].

3. Triplectic quantization

The main idea of the triplectic quantization proposed by Batalin, Marnelius and Semikhatov [5] was to consider fields π^{Aa} , which appear in the $Sp(2)$ -method, as anticanonical partners to the antifields $\bar{\phi}_A$ about extended antibrackets defined by the rule:

$$(F, G)^a \equiv \left(\frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi_{Aa}^*} + \epsilon^{ab} \frac{\delta F}{\delta \pi^{Ab}} \frac{\delta G}{\delta \bar{\phi}_A} \right) - (F \leftrightarrow G) (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}. \quad (17)$$

The new extended antibrackets have properties which formally coincide with the properties of the extended antibrackets within the $Sp(2)$ -formalism (see, for example, (5)). In an analogous way the operators V^a , Δ^a are introduced by

$$V^a = \frac{1}{2} \left(\varepsilon^{ab} \phi_{Ab}^* \frac{\delta}{\delta \bar{\phi}_A} - (-1)^{\varepsilon_A} \pi^{Aa} \frac{\delta_l}{\delta \phi^A} \right), \quad (18)$$

$$\Delta^a = (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi_{Aa}^*} + (-1)^{\varepsilon_A+1} \epsilon^{ab} \frac{\delta_l}{\delta \pi^{Ab}} \frac{\delta}{\delta \bar{\phi}_A}. \quad (19)$$

It can be readily established that the algebra of operators (18), (19) has the form

$$V^{\{a} V^{b\}} = 0, \quad \Delta^{\{a} \Delta^{b\}} = 0, \quad (20)$$

$$\Delta^a V^b + V^b \Delta^a = 0. \quad (21)$$

Note that the definition (18) of the new operators V^a differs from the $Sp(2)$ -case (see Eq.(6)). As a consequence, formulas (21) are valid without symmetrization in the indices a and b in comparison with the $Sp(2)$ -formalism. It is also usefully to introduce an operator $\bar{\Delta}^a = \Delta^a + (i/\hbar)V^a$ with the properties

$$\bar{\Delta}^{\{a} \bar{\Delta}^{b\}} = 0. \quad (22)$$

The vacuum functional in this approach is defined by the rule

$$Z(0) = \int d\phi d\phi^* d\pi d\bar{\phi} d\lambda \exp \left\{ \frac{i}{\hbar} (S + X) \right\} \quad (23)$$

where the boson functional $S = S(\phi, \phi^*, \pi, \bar{\phi}; \hbar)$ satisfies the following master equations

$$\bar{\Delta}^a \exp \left\{ \frac{i}{\hbar} S \right\} = 0. \quad (24)$$

or, equivalently,

$$\frac{1}{2} (S, S)^a + V^a S = i\hbar \Delta^a S, \quad (25)$$

and the boson functional $X = X(\phi, \phi^*, \pi, \bar{\phi}, \lambda; \hbar)$ is a hypergauge fixing action depending on new variables λ^A , $\epsilon(\lambda^A) = \epsilon_A$ and satisfying the following quantum equations:

$$\frac{1}{2} (X, X)^a - V^a X = i\hbar \Delta^a X, \quad (26)$$

which differs from Eq.(25) by the opposite sign of the V-term.

The vacuum functional (23) possesses the important property of invariance under the following global transformations

$$\delta\mathcal{G} = (\mathcal{G}, -S + X)^a \mu_a + 2\mu_a V^a \mathcal{G}, \quad (27)$$

where \mathcal{G} denotes the complete set of variables and μ_a is an $\text{Sp}(2)$ doublet of constant anticommuting Grassmann parameters. These transformations realize in the triplectic quantization the extended BRST transformations in the space of the variables $\phi, \phi^*, \bar{\phi}, \pi^a$ and λ .

If we consider the transformations (27) with μ_a depending on \mathcal{G} and λ it is not difficult to obtain the following representation for the vacuum functional

$$Z(0) = \int d\mathcal{G} \exp \left\{ \frac{i}{\hbar} \left[S + X - i\hbar(\mu_a, S)^a + i\hbar(\mu_a, X)^a + 2i\hbar V^a \mu_a \right] \right\} \quad (28)$$

Let us make an additional change of variables in the integral (28)

$$\delta\mathcal{G} = \frac{1}{2}(\mathcal{G}, \delta F_a)^a. \quad (29)$$

This change gives

$$\begin{aligned} Z(0) = \int d\mathcal{G} \exp \left\{ \frac{i}{\hbar} \left[S + X - i\hbar(\mu_a, S)^a + i\hbar(\mu_a, X)^a + \right. \right. \\ \left. \left. + 2i\hbar V^a \mu_a + \frac{1}{2}(S, \delta F_a)^a + \frac{1}{2}(X, \delta F_a)^a - i\hbar \Delta^a \delta F_a \right] \right\} \end{aligned} \quad (30)$$

If we identify

$$\delta F_a(\mathcal{G}) \equiv \frac{2\hbar}{i} \mu_a(\mathcal{G}, \lambda) \quad (31)$$

then we obtain

$$Z(0) = \int d\mathcal{G} \exp \left\{ \frac{i}{\hbar} \left[S + X + \delta X \right] \right\} \quad (32)$$

where the following notation has been introduced

$$\delta X = (X, \delta F_a)^a - V^a \delta F_a - i\hbar \Delta^a \delta F_a. \quad (33)$$

One can now check (for details, see [5])

$$(X, \delta X)^a - V^a \delta X = i\hbar \Delta^a \delta X \quad (34)$$

provided δF_a is chosen to have the following form

$$\delta F_a = \epsilon_{ab} \left\{ (X, \delta Y)^b - V^b \delta Y - i\hbar \Delta^b \delta Y \right\}. \quad (35)$$

On the other hand, any small admissible variation of hypergauge fixing action δX in Eq. (23) has to satisfy Eqs. (34). It means that one can compensate for a variation of hypergauge fixing action in the vacuum functional by a suitable choice of δF_a in (29) (or δY in (35)). Therefore, the vacuum functional (23) does not depend on the gauge. We see that from the formal point of view the triplectic quantization [5] possesses all remarkable features of the $Sp(2)$ -method.

4. Modified triplectic quantization

Notice that in its original version [5], the classical action $S_0(A)$ does not provide a solution of the master equations (25) because of the special structure of operators V^a (18) and cannot be considered as boundary condition to the master equations. On the other hand all known schemes of covariant quantization and existence theorems for them are based on the fact that all information of initial classical system is introduced through the boundary condition. This point should be considered as essential part of covariant quantizations.

We will show that it is possible to modify the original form of triplectic quantization [5] preserving all attractive features of the method and allowing to encode in the usual way any information about the classical system with the help of the boundary condition to the master equations.

We use the definition of extended antibrackets given in (17). In solving the functional equations determining the effective action we make use of the operators Δ^a , V^a and U^a

$$\Delta^a = (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi_{Aa}^*} + (-1)^{\varepsilon_{A+1}} \varepsilon^{ab} \frac{\delta_l}{\delta \pi^{Ab}} \frac{\delta}{\delta \bar{\phi}_A}, \quad (36)$$

$$V^a = \varepsilon^{ab} \phi_{Ab}^* \frac{\delta}{\delta \bar{\phi}_A}, \quad (37)$$

$$U^a = (-1)^{\varepsilon_{A+1}} \pi^{Aa} \frac{\delta_l}{\delta \phi^A}. \quad (38)$$

Notice that the operators Δ^a have already appeared both within the scheme of triplectic quantization [5] and, virtually, within the scheme of superfield quantization [4]. Even though the operators V^a in eq. (37) differ from the corresponding operators of the triplectic quantization, they coincide, at the same time, with the operators applied in the framework of the $Sp(2)$ method [3]. The use of the operators U^a in eq. (38) (an analog of these operators has been introduced in the method of superfield quantization) exhibits an essentially new feature as compared to both the $Sp(2)$ -method and the ‘old’ triplectic quantization.

One readily establishes the following algebra of the operators (36)–(38):

$$\begin{aligned} \Delta^{\{a} \Delta^{b\}} &= 0, \quad V^{\{a} V^{b\}} = 0, \quad U^{\{a} U^{b\}} = 0, \\ \Delta^{\{a} V^{b\}} + V^{\{a} \Delta^{b\}} &= 0, \quad \Delta^{\{a} U^{b\}} + U^{\{a} \Delta^{b\}} = 0, \\ V^a U^b + U^b V^a &= 0, \quad \Delta^a V^b + V^b \Delta^a + \Delta^a U^b + U^b \Delta^a = 0. \end{aligned} \quad (39)$$

Apart from Δ^a , V^a , U^a , we also introduce the operators $\bar{\Delta}^a \equiv \Delta^a + (i/\hbar)V^a$, $\tilde{\Delta}^a \equiv \Delta^a - (i/\hbar)U^a$. With the above mentioned properties it follows that the algebra of these operators has the form

$$\bar{\Delta}^{\{a} \bar{\Delta}^{b\}} = 0, \quad \tilde{\Delta}^{\{a} \tilde{\Delta}^{b\}} = 0, \quad \bar{\Delta}^{\{a} \tilde{\Delta}^{b\}} + \tilde{\Delta}^{\{a} \bar{\Delta}^{b\}} = 0. \quad (40)$$

Let us denote by $S = S(\phi, \phi^*, \pi, \bar{\phi})$ the quantum action, corresponding to the initial classical theory with the action $S_0 = S_0(A)$, and defined as a solution of the following master equations:

$$\bar{\Delta}^a \exp \left\{ \frac{i}{\hbar} S \right\} = 0. \quad (41)$$

with the standard boundary condition

$$S|_{\phi^*=\bar{\phi}=\hbar=0} = S_0. \quad (42)$$

Let us further define the vacuum functional as the following functional integral ($\mathcal{G} = (\phi, \phi^*, \pi, \bar{\phi}, \lambda)$):

$$Z = \int d\mathcal{G} \exp \left\{ \frac{i}{\hbar} (S + X + \phi_{Aa}^* \pi^{Aa}) \right\}, \quad (43)$$

where $X = X(\phi, \phi^*, \pi, \bar{\phi}, \lambda)$ is a bosonic functional depending on the new variables λ^A , $\varepsilon(\lambda) = \varepsilon_A$, which serve as gauge-fixing parameters. We require that the functional X satisfies the following master equation:

$$\tilde{\Delta}^a \exp \left\{ \frac{i}{\hbar} X \right\} = 0. \quad (44)$$

Notice that the generating equations determining the quantum action S in eq. (41) and the gauge-fixing functional X in eq. (44) differ – along with the vacuum functional Z in eq. (43) – from the corresponding definitions applied in the method of triplectic quantization [5].

One can readily obtain the simplest solution of eq. (44) determining the gauge-fixing functional X

$$\begin{aligned} X &= \left(\bar{\phi}_A - \frac{\delta F}{\delta \phi^A} \right) \lambda^A - \frac{1}{2} \varepsilon_{ab} U^a U^b F = \\ &= \left(\bar{\phi}_A - \frac{\delta F}{\delta \phi^A} \right) \lambda^A - \frac{1}{2} \varepsilon_{ab} \pi^{Aa} \frac{\delta^2 F}{\delta \phi^A \delta \phi^B} \pi^{Bb}, \end{aligned} \quad (45)$$

where $F = F(\phi)$ is a bosonic functional depending only on the fields ϕ^A . As a straightforward exercise, one makes sure that the functional X in eq. (45) does satisfy eq. (44). If we further demand that the quantum action S does not depend on the fields π^A , then the functional (43) becomes exactly the vacuum functional of the $Sp(2)$ quantization scheme [3].

Let us consider a number of properties inherent in the present scheme of triplectic quantization. In the first place, the vacuum functional (43) is invariant under the following transformations:

$$\delta \mathcal{G} = (\mathcal{G}, -S + X)^a \mu_a + \mu_a (V^a + U^a) \mathcal{G}, \quad (46)$$

where μ_a is an $Sp(2)$ doublet of constant anticommuting parameters. The transformations (46) play the role of the transformations of extended BRST symmetry, realized on the space of the variables $\mathcal{G} = (\phi, \phi^*, \pi, \bar{\phi}, \lambda)$.

Consider now the question of gauge dependence in the case of the vacuum functional Z (43). Any admissible variation δX should satisfy the equations

$$(X, \delta X)^a - U^a \delta X = i\hbar \Delta^a \delta X. \quad (47)$$

It is convenient to consider an $Sp(2)$ -doublet of operators $\hat{S}^a(X)$, defined by the rule

$$(X, F)^a \equiv \hat{S}^a(X) \cdot F, \quad (48)$$

and possessing the properties

$$\hat{S}^{\{a}(X) \hat{S}^{b\}}(X) = \hat{S}^{\{a} \left(\frac{1}{2} (X, X)^{b\} \right), \quad (49)$$

which follow from the generalized Jacobi identities (5). Consequently, eq. (47) can be represented in the form

$$\hat{Q}^a(X) \delta X = 0, \quad (50)$$

where we have introduced an $Sp(2)$ -doublet of nilpotent anticommuting operators \hat{Q}^a , defined by the rule $\hat{Q}^a(X) = \hat{S}^a(X) - i\hbar\tilde{\Delta}^a$, $\hat{Q}^{\{a}(X)\hat{Q}^{b\}}(X) = 0$. Then any functional of the form

$$\delta X = \frac{1}{2}\varepsilon_{ab}\hat{Q}^a(X)\hat{Q}^b(X)\delta Y, \quad (51)$$

with an arbitrary bosonic functional δY , is a solution of eq. (50). Moreover, by analogy with the theorems proved in Ref. [3], one establishes the fact that any solution of eq. (50) – vanishing when all the variables in δX are equal to zero – has the form (51), with a certain bosonic functional δY .

Let us denote by $Z_X \equiv Z$ the value of the vacuum functional (43) corresponding to the gauge condition chosen as a functional X . In the vacuum functional $Z_{X+\delta X}$ we first make the change of variables (46), with $\mu_a = \mu_a(\mathcal{G}, \lambda)$, and then, accompanying it with a subsequent change of variables $\delta\Gamma = (\mathcal{G}, \delta Y_a)^a$, $\varepsilon(\delta Y_a) = 1$, with $\delta Y_a = -i\hbar\mu_a(\mathcal{G}, \lambda)$, we arrive at

$$Z_{X+\delta X} = \int d\mathcal{G} \exp \left\{ \frac{i}{\hbar} \left(S + X + \delta X + \delta X_1 + \Phi_{Aa}^* \pi^{Aa} \right) \right\}. \quad (52)$$

In eq. (52) we have used the notation

$$\delta X_1 = 2 \left((X, \delta Y_a)^a - U^a \delta Y_a - i\hbar \Delta^a \delta Y_a \right) = 2\hat{Q}^a(X)\delta Y_a. \quad (53)$$

Let us choose the functional δY_a in the form

$$\delta Y_a = \frac{1}{4}\varepsilon_{ab}\hat{Q}^b\overline{\delta Y}, \quad \varepsilon(\overline{\delta Y}) = 0. \quad (54)$$

Then, representing δX as in eq. (51), and identifying $\delta Y = -\overline{\delta Y}$, we find that

$$Z_{X+\delta X} = Z_X, \quad (55)$$

i.e., the vacuum functional (and also the S matrix) does not depend on the choice of gauge.

5. Discussion

The reader may profit by considering the original version of triplectic quantization [5] as compared to the modified scheme, proposed in [12]. Thus, both versions are based on extended BRST symmetry. Both versions apply the vacuum functional and the S matrix not depending on the choice of gauge. Both versions implement the idea of separate treatment of the quantum action and the gauge-fixing functional, based each on appropriate master equations. The principal distinctions concern a different form of these equations as well as a different form of the vacuum functional. The modification of the generating equations [5] permits incorporating the information contained in the initial classical action by means of the corresponding boundary conditions. In contrast to the original version [5], the classical action provides a solution of the modified master equation. Thus, one establishes a connection with the previous schemes of covariant quantization. In particular, one easily reveals the fact of equivalence with the $Sp(2)$ quantization, by means of explicit realization of the corresponding class of boundary condition. In the original version of triplectic quantization, however, these questions still remained open.

Another distinction of the two triplectic quantization schemes is connected with the explicit structure of the corresponding master equations. Thus, the original version [5] of triplectic quantization defined the generating equations for the quantum action and the vacuum functional, using the operators

$$V_{\text{BM}}^a = \frac{1}{2} \left(\varepsilon^{ab} \phi_{Ab}^* \frac{\delta}{\delta \bar{\phi}_A} + \pi^{Aa} (-1)^{\varepsilon_A+1} \frac{\delta_l}{\delta \phi^A} \right) = \frac{1}{2} (U^a + V^a). \quad (56)$$

The use of the generating equations determining the quantum action with the help of the operators V_{BM}^a leads to the following characteristic feature of the triplectic quantization [5]: the classical action of the initial theory, defined as a limit of the quantum action at $\hbar \rightarrow 0$ and $\phi^* = \bar{\phi} = \pi = 0$, does not satisfy the generating equations of the method. In turn, the proofs of the existence theorems for the generating equations in all known methods of Lagrangian quantization are based on the fact that the initial classical action is a solution of the corresponding master equations. Moreover, from the viewpoint of the superfield quantization [4], which applies operators V^a , U^a , whose component representation is

$$\begin{aligned} V^a &= \varepsilon^{ab} \phi_{Ab}^* \frac{\delta}{\delta \bar{\phi}_A} - J_A \frac{\delta}{\delta \Phi_{Aa}^*}, \\ U^a &= (-1)^{\varepsilon_A+1} \pi^{Aa} \frac{\delta_l}{\delta \phi^A} + (-1)^{\varepsilon_A} \varepsilon^{ab} \lambda^A \frac{\delta_l}{\delta \pi^{Ab}} \end{aligned} \quad (57)$$

(with J_A being the sources to the fields ϕ^A), the operators (56) have no precise geometrical meaning, whereas the V^a and U^a in eq. (57) serve as generators of supertranslations – in superspace spanned by superfields and superantifields – along additional (Grassmann) coordinates. In turn, the operators V^a (37) and U^a (38) can be considered as limits (at $J_A = 0$, $\lambda^A = 0$) of the operators (57), which possess a clear geometrical meaning.

The present modified scheme of triplectic quantization enjoys every attractive feature of the quantization [5]: the theory possesses extended BRST transformations; the vacuum functional and the S matrix do not depend on the choice of the gauge-fixing functional; there exists such a choice of the gauge-fixing functional and solutions of the generating equations that reproduces the results of the $Sp(2)$ method.

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